The Modeling and Optimization of Building the Multi-dam System on Zambezi River

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Abstract

In this paper, we mainly provide a proper maintenance plan for the Kariba Dam in Africa which falls into disrepair and is facing to collapse. Firstly, we make a threshold analysis of the three options about their costs which include people's moving, old dam's removing, new dams' building, later repairing, ecological destruction and their incomes which include generation energy, avoiding of flood disasters' loss, providing employment, tourism resources and ecological protection. Then we get the specific relationship between benefits and years with some collected data. Both of the results show that the third option is the best choice from the economic view. And the result is completely as same as the conclusion we get after studying deeply on Option 3. Secondly, we regard water management capabilities as the safety coefficient of dams. We select 30 seed points along the riverbank for preparing the establishment of dams. With flow-between-riverway model, Manning equations, large Cauchy distribute function we get the scores of the seed points. We give an advice that the number of dams should be more and the positions of dams should be well-distributed. Then, we build an assessment model by analytic hierarchy process. We select three factors among all the factors, safety, economy and population. After testing the consistency, we get the weights of each factor: 0.6442, 0.2705, 0.0852. Then we value the factors and get an optimal scheme during the assessment with 0-1 integer programming: the number of dams is 17 and the longitude and latitude of them are shown in Table 17. The sensitivity of the result is tested as well. We also provide some strategies for the managers of ZRA to use. We suggest that they should use the dams normally in general. With the Dam-break model, we find 13 points among 17 points which are shown in Table 20. The dams at the 13 points need to be closed when there is a flood and it is just the opposite when the drought happens. For the extreme water flow, we assume an ideal water flow at first. The extreme water flow has to be adjusted to satisfy the ideal one. As for the restrictions in extreme conditions, the biggest impact happens at the 8th point among the 17 points. If the duration of maximum flow is t_0, the drainage time t to make the water flow return to the normal level equals to $4.95t_0$.

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1 Introduction

In section one, we will talk about the background of the problem and introduce our work briefly. The restatement of the problem will be shown in this part as well.

1.1 Background

The Kariba Dam is a double curvature concrete arch dam in the capital of Zambia. The scene and location are shown in figure 1 and figure 2. It was constructed between 1955 and 1959 at a cost of 135,000,000 dollars for the first stage.

The project is divided into two phases of construction, the one is the right bank in Zimbabwe while the other is the left bank in Zambia. The dam has made great contributions to people in Zambia and Zimbabwe, not only stored flood for combating the drought and preventing the water disaster, but also provided them a large amount of electric power. It is one of the world's largest dam which total reservoir capacity is 184 billion m^3. The installed capacity of power station is 1 million 500 thousand kW and the annual power generation is up to 8 billion kW/h. Besides that, the dam is now one of the most famous tourist attractions in Zambia. Many tourists are attacked by The Kariba dam every year.

But the dam also brings a lot risks. According to the report, Impact of the failure of the Kariba dam, made by The Institute of Risk Management South Africa Risk Research Report in 2015[3], It has a potential danger of collapse. If the dam collapsed, it would threaten the lives and property of the about 3500000 residents of four countries and the power supply in the whole region. It also brings other crisis in economics, environment, politics (for it crosses into many countries) and sociology. We must find a solution to solve the problem of the Kariba dam.

1.2 Our Work

In order to provide help for the Zambezi River Authority (ZRA), to provide some scientific reference for the actual construction of the dam. We collected a large number of data and build different models to do the following research. The previous researchers did not discuss that use a series of small dams to replace the large dam in depth. In this paper, we will discuss this issue deeply.

First of all, we make a horizontal comparison among the three options and make qualitative comparison and quantitative comparison. Provide a scheme preliminarily.

Then, we find the relationship between Q and $\sqrt{S}$ (the flow and the square root of the slope) by flow-between-riverway model from two aspects that are water management capabilities and the water management capabilities. And we can find the specific formula of Q with data. Assign and evaluate them to get the conclusion and suggestions. We select three relatively important factors (safety, economy and population) among all the factors. We build the model and calculate the weight of each factor. The consistency of the weights are checked and each factor is evaluated.

After that, we provide a strategy for the managers of ZRA, including how to use the series of dams under different conditions of normal, dry and flood by the dam-break model. We also

give the values of extreme flow under the normal condition and the limiting conditions under different conditions.

Finally, we analyse the advantages and disadvantages of the model.

In order to make our thinking clearly, we make a map of our work that is shown in figure3.

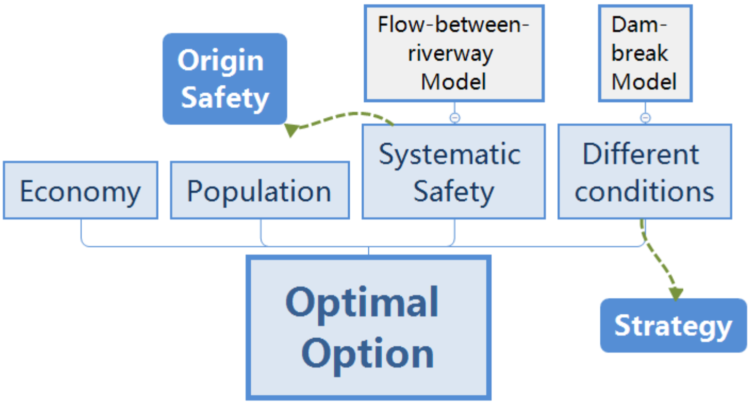

Figure 1: a map of our work

1.3 Problem Restatement

The problems that we need to solve in this paper are :

- Develop a cost-benefits model about different factors that influence the assessment of the final choice. Provide a brief assessment of the three options (repairing the old one, rebuilding the old one or replacing with a series of smaller dams) to ZRA with enough supporting details about potential costs and benefits.
- Analyse the way that replacing the old dam with a series of smaller dams in detail and give a proper strategy. Use the model you build to ensure a way that consider the safety and costs, the maximum and minimum expected water flows, the emergency circumstances, the proper amount and positions of small dams etc. Present your strategy.

2 Preparation

We will give the general assumptions and the symbols of the whole passage for preparation.

2.1 General Assumptions

- The water flow is one-dimensional. That is to say that the water flow is in the two-dimensional plane which is simplified by the three-dimensional sphere and degree of freedom is one.
- We don't consider about the vertical flow and the vertical acceleration.
- Water can not be compressed.
- The river tributaries are small. We assume that they have no effect on the main channel.

- The cross-sectional area and the wet perimeter of each reach are constant.
- The friction effect of water flow is the same as that of uniform flow.
- Dense discrete reach points show the curve characteristics of reaches.
- Normally, the slope of the river is only related to the slope of riverbed.

2.2 Symbols

Table 1: Symbols

Symbols	Descriptions
Q	the water flow
E	the benefit
ω	the cross-sectional area of water
n	amount of the dams
c	Chezy's coefficient
R	hydraulic radius
J	hydraulic gradient
V	the volume of water
u	average flow velocity
n_m	coefficient of frictional resistance of channel
A	the area of cross section
P	wetted perimeter
S_i	the slope of reaches
d	the depth of water
x	the direction of propagation
t	time
g	the acceleration of gravity
S_f	the slope of friction
S_o	the slope of riverbed
l	the curve distance along the river bank from the origin
N_p	the population along the river
β	the safety coefficient of dam
β_s	the systematic safety coefficient
β_i	the individual safety coefficient
σ	the standard deviation of distance difference
$\bar{d}$	the average value of distance difference

3 A Brief Assessment of Three Options

The different sizes of the dam have their own advantages and disadvantages. In order to find the optimal scheme, we need to evaluate the advantages and disadvantages from diversity sides of them (L=large; M=medium; S=small). Taking into account the realities of Africa, such as climate, economy and so on. We can get the results of qualitative comparison below in Table 2 (the assessment on costs of three options) and Table 3 (the assessment on benefits of three options).

In the qualitative judgement of the costs, we can only use ”large”,”medium” and ”small” to evaluate and compare the three options with relative standards. In the estimated cost, we can

Table 2: the Assessment on Costs of Three Options

	People's Removing	Old Dam's Removing	New Dam's Building	Later Repairing	Ecological Destruction
I	S	S	S	L	S
II	M	M	M	M	M
III	L	L	L	S	L

compare them from five aspects: people's removing, old dam's removing, new dam's building, later repairing and ecological destruction.

It can be obtained obviously, if we only focus on the cost of construction, the first option (i.e. the renovation of dam on the basis of the original) has the lowest cost, the third option (i.e. the demolition of the original dam and build a series of small dams) has the highest cost. The next step for us is to continue to make qualitatively judgement on the benefits of the three options so that we can make a further and correct judgement.

Table 3: the Assessment on Benefits of Three Options

	Generation Energy	Avoiding of Flood Disaster's Loss	Provide Employment	Provide Tourism Resources	Ecological Protection
I	M	S	S	S	S
II	M	M	M	M	M
III	L	L	L	L	L

We compare the benefits qualitatively of the three options from generation energy, avoiding of flood disaster's loss, provide employment, provide tourism resources and ecological protection. Through the estimated earnings, we can get the conclusion that the third option's cost is high , but the benefits in the future of all aspects are very large. As for the first option, although the cost is low, the income in the future is very low.

The following will be considered with both the cost and benefits of the three options for the final assessment. We ignored some aspects,for their spendings and incomes are not easy to measure with money. Luckily, those aspects only make small impacts. In the next data calculation, we only consider about the amount of money which is relatively large. Table 4 is a table for some specific calculation of several factors.

Table 4: Specific Calculation of Several Factors

	Building	People's Removing	Power Generation	Maintenance
I	\$294 million	0	1830 MW	0.5 year
II	\$3000 million	\$50 million	3000 MW	1 year
III	\$100 million$\times n$	\$50 million	300 MW $\times n$	1 year

$$\begin{cases} E = -|Cost| + |Benifits| \\ Benefits = Power\ generation \cdot Mainenance \\ Cost = Building + Immigrant \end{cases} \tag{1}$$

so we get

$$E = -Building - Immigrant + Power \cdot Mainenance \tag{2}$$

and we let

$$\bar{E} = 10^{-8} \cdot E \tag{3}$$

$$\begin{cases} \bar{E}_1 = -2.94 - 0 + 12.13 \times 0.5 = 3.125 \\ \bar{E}_2 = -30 - 0.5 + 19.88 \times 1 = -10.62 \\ \bar{E}_3 = -1 - 0.5 + 1.99n \times 1 = 1.99n - 1.5 \end{cases} \tag{4}$$

So we get $\bar{E}_3 > \bar{E}_1 > \bar{E}_2$ while $n \geq 10$.

Through the part above ,with the quantitative calculation, we can know that consider both costs and benefits, the third option is the best choice. It also verifies the results which obtained from the qualitative analysis.

We also collect some data to support our option. we list the specific data and their category in table 5.

Table 5: the Category of the Data

Option 1	cost for maintenance	\$294 million
	cost for people's moving	\$0
	benefit for generating electricity	1830MW
Option 2	cost for reconstruction	\$3000 million
	benefit for generating electricity	3000MW
	the moving population	46,000
Option 3	cost for single dam's construction	300 MW
	benefit for single dam's generating electricity	\$100 million

So, it can be seen that **the third option** is the best choice from an economic point of view. This conclusion is exactly the same as that we get from the deep study on the third option below.

4 Analysis of Option 3

According to the option 3, we have to give a strategy for building small dams. The points have to satisfy some factors, as water management capabilities and water management options.

In order to facilitate study, we take the original position of the dam as the origin of distance and collect **30 seed points** on the reach we selected ($252km$) uniformly, based on the points of extreme value of the slope. We take "kilometres" as the scale of distance. The Rome numbers

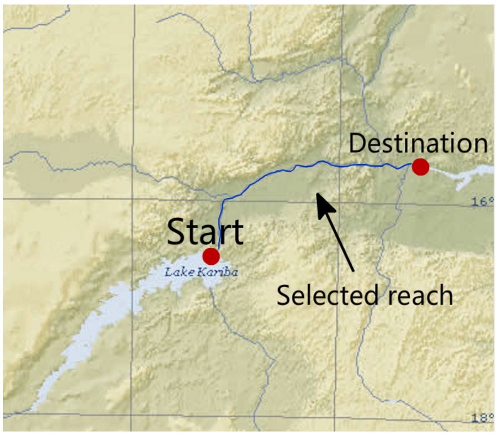

Figure 2: the Reach We Selected

in the tables represent different points. The corresponding relation among the Roman numbers, the abstract distance and actual distance is shown in table 6 below (The distance in the following tables are the abstract distance).

Table 6: Actual Distance (km) and Rome Digital Corresponding Point

	I	II	III	IV	V	VI	VII	VIII	IX	X
Abstract	1	2	3	4	5	6	7	8	9	10
Actual	11.33	26.69	35.33	42.91	55.33	63.30	67.47	75.69	79.21	83.44
	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Abstract	11	12	13	14	15	16	17	18	19	20
Actual	87.50	95.98	103.69	109.35	115.25	127.72	137.20	142.77	151.77	157.79
	XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII	XXIX	XXX
Abstract	21	22	23	24	25	26	27	28	29	30
Actual	169.62	176.19	184.11	189.06	194.31	200.11	209.11	215.71	220.84	226.01

4.1 Analyse Water Management Capabilities

We will talk about the water management capabilities below. The water management capabilities are positively correlated to safety which is related to the slope of the river. Figure 4 is the terrain profile of basins with cities and we can know the slope of each reach directly from the treated data.

4.1.1 Determine the Relationship Between Flow and Slope by Flow-Between-Riverway Model

The safety of people's lives and property is the most important thing and that is the original reason why the relevant departments establish the dam. The safety of the dam is closely related to the flow of the river. Dams need to be built where they are needed (always dangerous places). Therefore, we divide the rivers into different sections, and set up the **flow-between-riverway model**.

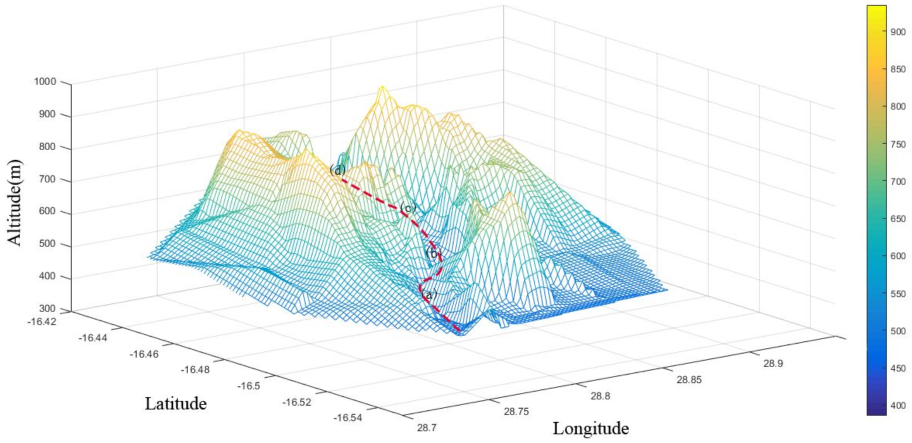

Figure 3: the Terrain Profile of the Basins With Cities

In the following discussion, we find some factors that influence the arrangement and number of small dams.

To build our model, the next step is calculating the river flow by **Manning Equation**.

$$\begin{cases} Q = \omega \cdot c\sqrt{R \cdot J} \\ \dfrac{\partial \rho}{\partial t} + \bigtriangledown \cdot (\rho u) = 0 \end{cases} \tag{5}$$

Through this model, we can estimate the volume of each reaches and the cross sectional area of downstream channel. Then, we use the Manning equation and the **continuity equation** to establish a set of **first-order differential equations** and get the differential equations for the numerical solution. Finally, we test the model with normal flow 's data from the hydrological observation made by the station.

To ensure the river's segment points, we try to choose the reaches which are relatively straight and narrow basin. We Assume that the flow of water is determined by the speed at narrow reaches. In this way, the entire river can be seen as a series of pools, which the water flow through the pools.

In general, the flood can be used Manning equation, uniform channel flow analysis method:

$$u = \frac{1}{n_m}(\frac{A}{P})^{\frac{2}{3}}\sqrt{S} \tag{6}$$

Among them, u is the average velocity of water flow. n_m is the coefficient of friction resistance of reaction channel. A is the cross section area of the channel. P is the wetted perimeter(the circumference of the cross-sectional area of the flow). S is the slope of Channel.

There has been no theoretical basis for Manning equation so far. However, a large number of experiments have proved its existence and effectiveness. Its main advantage is that a large amount of information can be used to estimate **Manning roughness** n_m.

We Assume that the slope of channel S is equal to the slope of the riverbed. That is to say, for any two points, the direction of the flow has nothing to do with the depth of water. The slope

is calculated by dividing the elevation difference between two points by the horizontal distance. We use the data from a report made by Nugent(in 1990) to estimate the slope of each section.

$$S_1 = \frac{1}{237}, S_2 = \frac{1}{351}, S_3 = \frac{1}{423}, S_4 = \frac{1}{497}, S_5 = \frac{1}{948}$$

The terrain data of river can be used to estimate some amounts, including the volume of water, the area of the cross section, the wet perimeter and the height of the water. According to that, we can establish section i cross-sectional area of the river basin A_i and wetted perimeter P_i on the function of the volume of river water V. For given channel cross section, the flow Q satisfies the following formula:

$$Q = uA \tag{7}$$

u is the mean velocity of water flow. Put the result of(5) into (6):

$$Q_i = A_i u_i = \frac{A_i}{n_m} (\frac{A_i}{P_i})^{\frac{2}{3}} \sqrt{S_i} \tag{8}$$

In the formula above, the cross-sectional area A_i is $A_i(\rho V)$, the wetted perimeter P_i is $P_i(\rho V)$, the volume of river water V is $V(t - \zeta)$.

Two parameters, ρ and ζ are introduced to correct the model and the specific values can be obtained from the experimental data. ρ is the effect of the friction and surface characteristics of the channel on the downstream flow. ζ is the time requirement for water to flow through the channel. We can regard ζ as a constant because the length of each reaches are invariant.

4.1.2 Determine the Expression of Flow

Safety is the most essential factor at any time and under any circumstances. The safety coefficient of dam should be taken into account in determining the quantity of dam and the location of dam. The determination of the location of the dam is mainly in those places where flood protection and water retaining are needed, such as the large-flow areas. Moreover, the small dams should cooperate with each other. To determine the number of the dam, the dam is

Figure 4: Elevation of Zambezi River Basin

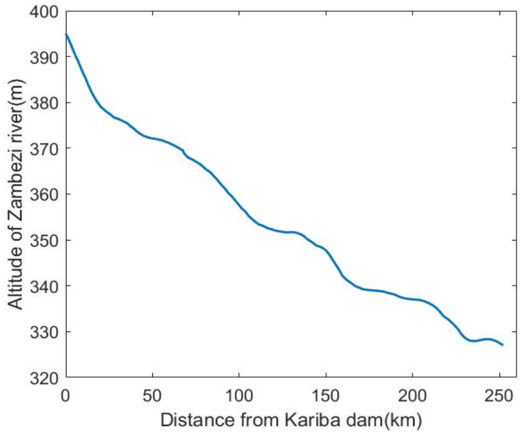

Figure 5: Channel elevation

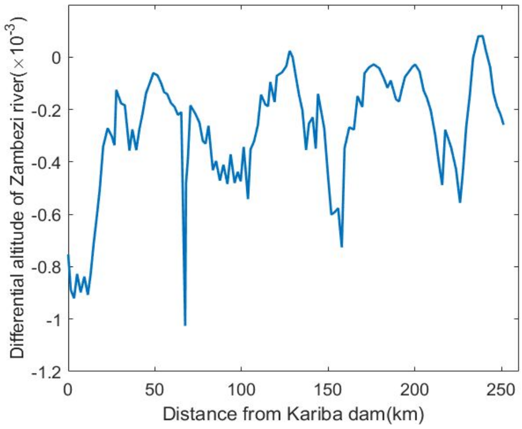

Figure 6: Channel Elevation Difference

comprehensive cost and the final effect. The purpose is to find a solution to ensure the safety of the premise, as far as possible to spend less.

We get some standard of the safety of dams and the differences between large dam and small dams from International dam Commission.

The safety coefficient of dam β is used to measure the degree of safety and it is determined by the individual safety coefficient β_i and the systematic safety coefficient β_s. The individual safety coefficient is related to the flow. The systematic safety coefficient is related to the average distance between two small dams and the degree of dispersion.

$$\beta = \frac{1}{2}(\beta_i + \beta_s) \tag{9}$$

Next, we will determine the individual safety coefficient by the flow. We can get the final expression of the flow Q from Manning Equation in the flow-between-riverway model. The elevation of Zambezi River Basin is shown in figure 6 and we can get the elevation of that area from the figure.

We assume the A_i , n_m , P_i are invariant.

$$Q \propto \sqrt{S_i} \tag{10}$$

$$Q = dh \tag{11}$$

The large flow is not good for dams building so we give the area with large flow low score during the process of normalized to five. The table below shows the specific data of flow that normalized to five β_i and distance l:

Using the data after normalized to five, we will quantify the three factors that influence the establishment of the small dams.

According to the qualities, we can judge them from bad to good. The membership degree can be valved from 0.01 to 5 that can be continuous. When the value becomes larger and larger, the effect of them become less and less significant. So we use the **large Cauchy distribute**

Table 7: Specific Data of Flow (Normalized to Five) and Distance

	I	II	III	IV	V	VI	VII	VIII	IX	X
Distance	1	2	3	4	5	6	7	8	9	10
Flow	1.24	2.71	2.64	3.17	3.55	3.15	1.00	3.02	2.73	2.41

	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Distance	11	12	13	14	15	16	17	18	19	20
Flow	2.29	2.26	2.09	2.98	3.29	5.00	2.65	2.67	1.94	1.63

	XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII	XXIX	XXX
Distance	21	22	23	24	25	26	27	28	29	30
Flow	3.28	4.34	3.65	3.42	3.92	4.34	3.22	2.24	2.68	2.05

function :

$$f(x) = \begin{cases} [1+\alpha(x-\eta)^{-2}]^{-1}, 1 \leq x \leq 3, \\ alnx+b, 3 < x \leq 5 \end{cases} \tag{12}$$

In the formula, α , η , a , b are undetermined constants.

So we can determine the values of the constants : $\alpha = 1.1086$, $\eta = 0.8942$, $a = 0.3915$, $b = 0.3699$. After putting them in (11), we can get the **membership function** :

$$f(x) = \begin{cases} [1+1.1086(x-0.8942)^{-2}]^{-1}, 1 \leq x \leq 3, \\ 0.3915lnx+0.3699, 3 < x \leq 5 \end{cases} \tag{13}$$

The graph of the function is shown below :

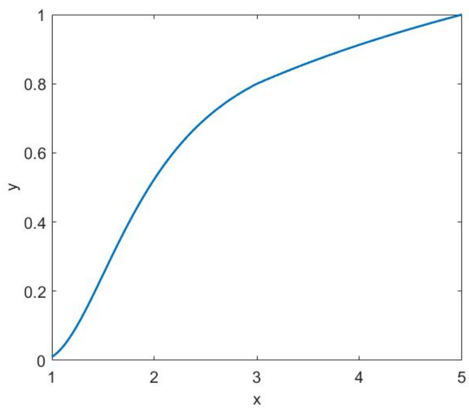

Figure 7: the Graph of the Membership Function

We treat the data above with the formula. The table below shows the specific data of flow that treated $\bar{\beta}_i$ and distance l. The ”Flow” means the score of flow.

The data above show the individual safety coefficient.

Then, we will determine the systematic safety coefficient. To make sure the coefficient, we have to talk about the average value of distance difference $\bar{d}$ and the standard deviation of distance difference σ. We use the geometric averages of them to estimate their relationship because β_s is positively correlated with $\bar{d}$ and σ.

Table 8: Specific Data of Flow (Treated) and Distance

	I	II	III	IV	V	VI	VII	VIII	IX	X
Distance	1	2	3	4	5	6	7	8	9	10
Flow	0.1	0.75	0.73	0.82	0.87	0.82	0.01	0.80	0.75	0.67
	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Distance	11	12	13	14	15	16	17	18	19	20
Flow	0.64	0.63	0.56	0.80	0.84	1.00	0.74	0.74	0.50	0.33
	XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII	XXIX	XXX
Distance	21	22	23	24	25	26	27	28	29	30
Flow	0.83	0.94	0.88	0.85	0.90	0.94	0.83	0.62	0.74	0.55

We assume that the dams will be established at some points among the thirty **seed points** (pre-establishing points). The amount of the dams is n. During the process of selecting, the points that selected by us are not necessarily continuous in the sequence of thirty points, so we express these selected points as x_{i_1} , x_{i_2} ... x_{i_n}. Similarly, from l_{i_1} to l_{i_n} is the distance from x_{i_1} (or x_{i_n}) to the origin. $\bar{d}$ is the average value of distance difference. The formula is shown:

$$\bar{d} = \frac{(l_{i_1} - l_{i_0}) + (l_{i_2} - l_{i_1}) + (l_{i_3} - l_{i_2}) + \ldots + (l_{i_n} - l_{i_{n-1}}) + (l_{i_{n+1}} - l_{i_n})}{n+1} \tag{14}$$

We have to utilize the standard deviation of distance difference to determine the degree of dispersion of the dams' location. The formula is shown :

$$\sigma = \sqrt{\frac{1}{n+1} \sum_{i=1}^{n+1} (\Delta l_i - \bar{d})^2} \tag{15}$$

With the data above, we can know the β_s :

$$\beta_s = \sqrt{\bar{d} \cdot \sigma} \tag{16}$$

We have a lot of options and each of them includes different amount and individual of dams. They can be judged from the best to the worst. We use β_s to judge them. We have to select the best option to use the **normalized algorithm**. However, we can not use the **enumeration method** because of too many samples, too long time and too many difficulties. So we use the **genetic algorithm** to select a better solution and then use the **theoretical analysis** to find the best solution. There is a positive correlation among the systematic safety, the standard deviation of distance difference and the average value of distance difference. So we can determine the interval of β_s by using the value of $\bar{d}$ and σ.

By the expression of $\bar{d}$ in (18), we can find that the denominator of $\bar{d}$ is the length of the section we selected, which is 250 km. So we put 250 km in the formula below directly.

The best option is that the amount of dams is the largest and they are put evenly. We can

get an endpoint of the final value interval from (19) according to the formula of β_s above:

$$\sigma = \sqrt{\frac{1}{11}\sum_{i=1}^{10}(0-\frac{250}{11})^2+\frac{1}{11}(250-\frac{250}{11})^2} = 40.7 \tag{17}$$

The worst option is that the amount of dams is the smallest and they are all put at the original point. For the value of standard deviation of distance difference is 0 under this circumstance, we get the other endpoint of it.

So **the final value interval** of β_s is from 0 to 40.7.

According to the normalization algorithm, we can get (”best” in the formula means the value of best condition, and similarly for ”worst”):

$$\bar{\beta}_s = 1-\frac{|\beta_s - best|}{best - worst} = 1-\frac{|\beta_s - 40.7|}{40.7-0} \tag{18}$$

Put the result into (9), we can get the relationship among normalized value of the safety coefficient $\bar{\beta}$, the systematic safety coefficient $\bar{\beta}_s$ and the individual safety coefficient $\bar{\beta}_i$:

$$\bar{\beta} = \frac{1}{2}(\bar{\beta}_i+\bar{\beta}_s) \tag{19}$$

According to the conclusion, we can make a deep study on the water management capabilities of original dam and the new systematic dams.

4.1.3 Compare the Effect between Original Dam and New Dams

In order for the new series of dams to have at least the same effect as the original dam, we will evaluate the effectiveness of the original dam and the small dams.

For the original dam, the flow is small at first and the slope of it is slow, so the score is high. The original dam do not have system, so there is no system safety coefficient. With the formula $\beta = \frac{1}{2}(\beta_s+\beta_i)$, the water management capabilities of the original dam is 0.796.

According to the table 8 which includes the mark of each ”seed point”, we select the seed points whose marks are in average top twenty.

Table 9: seed points in average top twenty

	I	II	III	IV	V	VI	VII	VIII	IX	X
Mark	0.733	0.735	0.739	0.742	0.748	0.752	0.797	0.803	0.818	0.822
	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Mark	0.828	0.835	0.836	0.851	0.866	0.877	0.905	0.945	0.945	1.000

Then, we consider the water management capabilities of small dams’ placement under extreme circumstance. To get the biggest water management capabilities, the **best** way is to build 20 small dams uniformly and the mark is 1. The **worst** way is to build 10 dams, that 5 of them are built at one endpoint of the reach while the others are built at the other endpoint of the reach. The mark is 0.32 in this condition.

We can also get conclusion that the mark will be higher if the amount and the average degree of building gets higher.

4.2 Analyse Water Management Options

To compare the water management options of the new systematic dams and the old dam, we explain it from two aspects, the length of each reach and the amount of options for selected.

- **The length of each reach** : The flow control regulation of multi-dam system can be more accurate than a single dam, which makes the water management of former is more efficient. The formula below show the principle. d is the deviation degree of flow control. l is the length of one reach. n is the amount of dams.

$$d = \frac{l}{n} < l \text{ , } n \geq 10 \tag{20}$$

- **The amount of options for selected** : Every dam has two status, open and closed. We give a characteristic value for each status as table 9.

Table 10: characteristic value for each status

Status	Open	Closed
Value	1	0

At the same time, according to the permutation and combination, when the amount of small dams is n, the amount of options is 2^n. For the amount of small dams are less than 10, we can get $2^n \geq 2^10 > 2^1 = 2$.

We can know that the water management options of **multi-dam system** is more than the single dam.

4.3 Give Suggestions for Positions and Amounts of Dams

We will choose the dam that have high individual score as possible. In addition, the location of them should be satisfied that the distribution is uniform on the selected reach (i.e. the dispersion degree of adjacent points is low).

5 Strategy for Practice

We will show the ZRA managers the specific strategy for the dam. The strategy have to include information about the specific amounts and positions of small dams and make consistency test. We also build a assessment to judge the different ”seed points” and find the most proper positions of them.

5.1 Scheme Balanced Safety and Costs(Analytic hierarchy process)

We want to build an assessment model to evaluate the fitness degree of different seed points with Analytic hierarchy process. We will provide a scheme that balanced the safety, the economy and the population.

5.1.1 Assessment Model of Dam Establishment and Weight of Each Factor

According to the actual situation ,we use **AHP (analytic hierarchy process)** to get the **analytic hierarchy process model**. Compare the importances of the three factors (Safety, Economy and Population) by 1-9 **pairwise comparison matrix** .

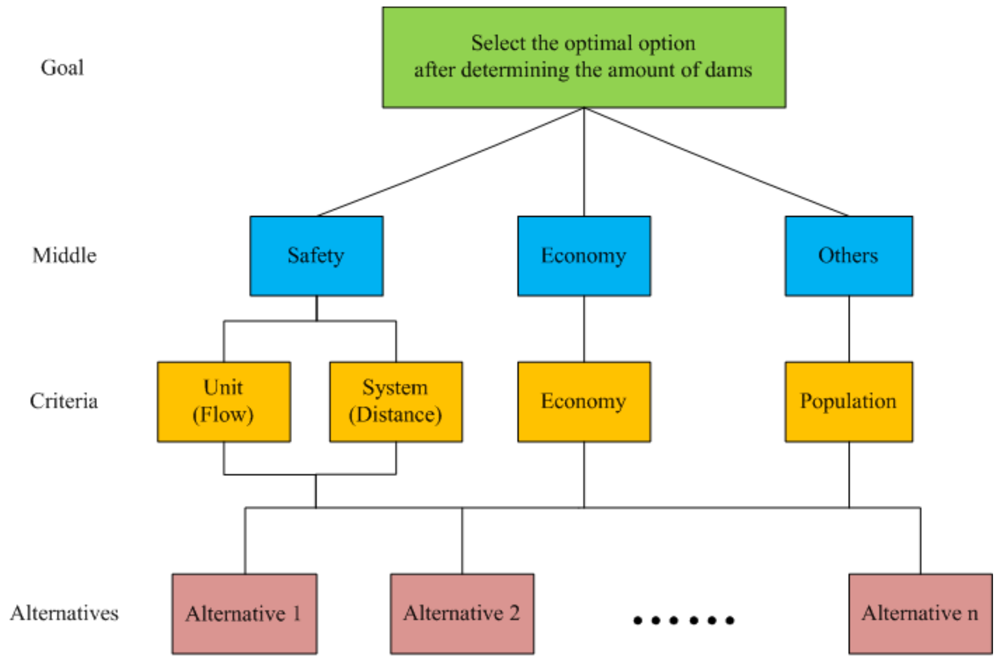

Figure 8: A Map of Analytic Hierarchy Process

When $a_{ij} = n$, n can only be valued from 1 to 9, $a_{ij} > 0$, $a_{ij} = 1$, $a_{ij} = \frac{1}{a_{ij}}$.

Table 11: the Relative Weight Value of the Pairwise Comparison Matrix

$a_{ij} = 1$	element i has the same importance as element j
$a_{ij} = 3$	element i is little essential than element j
$a_{ij} = 5$	element i is essential than element j
$a_{ij} = 7$	element i is more essential than element j
$a_{ij} = 9$	element i is much more essential than element j
$a_{ij} = 2n$	n=1,2,3,4 ; the importance of element i and element j lies between $2n-1$ and $2n+1$

For the dam building and finding the optimal option ,we consider three main factors : Safety, Economy and Population .We get the pairwise comparison matrix as below by **paired**

comparison.

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 6 \\ \frac{1}{3} & 1 & 4 \\ \frac{1}{6} & \frac{1}{4} & 1 \end{pmatrix} \tag{21}$$

Then, we will make a **consistency test** and get the weight of each factors.

5.1.2 Consistency Test of Model's Weight

Measure the degree of inconsistency of a contrast matrix A ($n > 1$ order matrix) is CI :

$$CI = \frac{\lambda_{max}(\mathbf{A}) - n}{n - 1} \tag{22}$$

(λ_{max} is the contrast matrix A 's greatest eigenvalue.)

After measuring, we get

$$\lambda_{max}(\mathbf{A}) = 3.0536$$

According to the formula, we measure the **consistency ratio** of pairwise comparison matrix A CR :

$$CR = \frac{CI}{RI} \tag{23}$$

We can get

$$CI = \frac{\lambda(\mathbf{A}) - 3}{3 - 1} = 0.0268$$

We can find the data from reference:

$$RI = 0.49$$

$$CR = \frac{CI}{RI} = \frac{0.0268}{0.49} = 0.0547 < 0.1$$

So,we can get the conclusion that A is consistent. At this time, the eigenvector of the maximum eigenvalue of A is :

$$U = (0.9152, 0.3844, 0.1211)$$

The weights of them are :

$$\bar{U} = (0.6442, 0.2705, 0.0852)$$

Finally, we get the **weights** of the three factors :

Table 12: Weights of the Three Factors

	Safety	Economy	Population
Weight	0.6442	0.2705	0.0852

From above, we know that A is **consistent** and the **weight of the three factors** are shown.

5.1.3 Value Based on Actual Conditions

We will value the three factors based on actual conditions : **Safety** , **Economy** and **Population**.

- **Safety**

The value of Safety has been calculated by us before (in 4.1 Water Management Capabilities). Results will not be repeated here.

- **Economy**

In the construction of the dam, the economy is also a very important factor to be considered. There is the cost of pre-investment, the cost of continuing-investment in the latter and the benefits in the future. In the pre-investment, the residents need to move away, the old dam need to be dismantled, the new dams should be established. In the continuing-investment, the dams should be maintained. The benefits in the future includes electricity generating, jobs providing, tourist resources. Under the condition of satisfying safety, the cost of inputs should be as little as possible, while the gains should be as much as possible. Economy is also a factor to limit the number and location of dams. Such as the number of established so that both safety standards, not waste. The determination of the amount and positions should satisfy the acquirement that cost less and gain more as far as possible.

We can find some data for calculation. The weight of economy is 0.2705 in the final assessment.

From the formula about $\bar{E}_3$ in the brief assessment of three options , we know that the amount of dams n is proportional to the benefits. We normalize the value of benefits E and the data are showed below. The "benefit" means the score of benefit.

Table 13: Specific Data of the Amount of Dams and Normalized Benefits

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
Amount	10	11	12	13	14	15	16	17	18	19	20
Benefit	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00

- **Population**

In this circumstance, there is a great relationship between the location of the dam and the population nearby. One of the factors to consider is the surrounding population. That is to say, the impact of building dams on the people should be considered. For example, the construction of the dam need the cooperation of the residents. They have to move out of the original location so that the water can be removed. The costs also need to be considered. In addition, the safety of dam is extremely essential in densely populated areas. Once the break of dam will threaten the safety of many people's lives and property.

We collected the data of the number of residents along the Zambezi River and that is the basis of calculation. The weight of population is 0.0852 in the final assessment.

According to the population density distribution map, the data of population (N_p) from the figure (only 15 and 37) were normalized. Show in the table below.

The area that population is intensive is not suitable for dams building because the dams may threaten people's safety and they need to move out. So the area will get lower score if it has more people in the evaluation. The "population" means the score of population.

Table 14: Specific Data of Flow and Distance

	I	II	III	IV	V	VI	VII	VIII	IX	X
Distance	1	2	3	4	5	6	7	8	9	10
Population	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.41	0.41	0.41

	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Distance	11	12	13	14	15	16	17	18	19	20
Population	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41

	XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII	XXIX	XXX
Distance	21	22	23	24	25	26	27	28	29	30
Population	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41

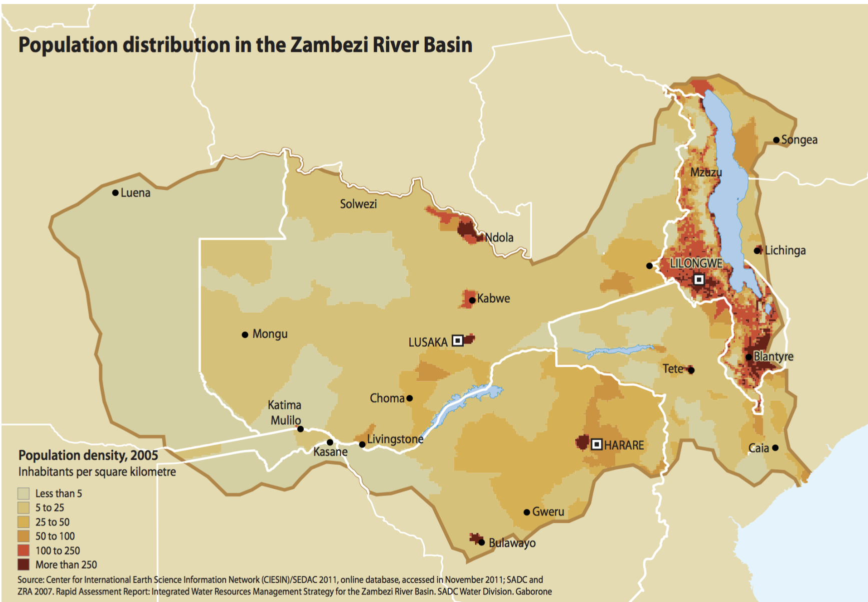

Figure 9: Population Distribution in the Zambezi River Basin

5.1.4 Final Optimal Option

Next, we will calculate the scores of all the schemes according to the value of each factor and their weights. We use the **0-1 integer programming** and **genetic algorithm** to find the best

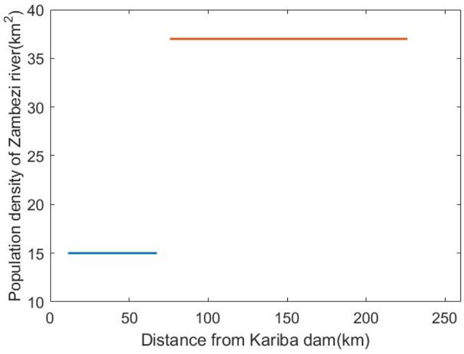

Figure 10: the Population along the Way of Zambezi River

solution. That is the best scheme that we want. Symbols in the formula are shown in table 12.

$$
\begin{aligned}
Score &= (s-v, e-v, p-v)\cdot(weight-vector)^T \\
&= (s-v, e-v, p-v)\cdot\begin{pmatrix} s-w \\ e-w \\ p-w \end{pmatrix} \\
&= (s-v, e-v, p-v)\cdot\begin{pmatrix} 0.6442 \\ 0.2705 \\ 0.0852 \end{pmatrix} \\
&= 0.6442(s-v)+0.2705(e-v)+0.0852(p-v)
\end{aligned}
\tag{24}
$$

Table 15: Some Symbols in the Table Above

Symbol	Description
s-v	value of safety
e-v	value of economy
p-v	value of population
s-w	weight of safety
e-w	weight of economy
p-w	weight of population

We find the specific position from x_{i_1} to x_{i_n} $(10 \leq n \leq 20)$. We want to get the minimum of $0.6442(s-v)+0.2705(e-v)+0.0852(p-v)$ from (12) such that

$$
\begin{cases}
10 \leq \sum\limits_{j=1}^{n} x_{i_j} \leq 20 \\
x_{i_j} = 0 \ or \ 1 \ (1 \leq j \leq n)
\end{cases}
\tag{25}
$$

According to 0-1 integer programming with genetic algorithm in Matlab and figure x , figure x and figure x, we can get that best option and its status is shown in table w below. ”1” in the table means that the position will be built a dam, ”0” is converse.

Table 16: the Determine of the Dams' Positions

	I	II	III	IV	V	VI	VII	VIII	IX	X
Distance	1	2	3	4	5	6	7	8	9	10
Status	1	1	1	1	1	1	1	1	0	0
	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Distance	11	12	13	14	15	16	17	18	19	20
Status	0	1	0	0	1	1	0	1	0	0
	XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII	XXIX	XXX
Distance	21	22	23	24	25	26	27	28	29	30
Status	1	0	1	0	0	1	1	0	0	1

From table x, we can know that the best scheme is to **build 17 dams**. Their positions in the river bank and longitude and latitude is shown in table x.

Table 17: the Latitude, the Longitude and the Distance of the Positions

	I	II	III	IV	V	VI	VII	VIII	IX
Distance	11.33	26.69	35.33	42.91	55.33	63.30	67.47	75.69	95.98
Latitude	-16.41	-16.29	-16.23	-16.16	-16.06	-16.00	-15.97	-15.94	-15.86
Longitude	28.84	28.83	28.84	28.84	28.85	28.88	28.90	28.96	29.11
	X	XI	XII	XIII	XIV	XV	XVI	XVII	
Distance	115.25	127.72	142.77	169.62	184.11	200.11	209.11	226.01	
Latitude	-15.76	-15.72	-15.69	-15.64	-15.61	-15.64	-15.63	-15.66	
Longitude	29.25	29.35	29.49	29.72	29.85	29.99	30.07	30.21	

Each item's score and total score of different points in our assessment is shown in table x. Their positions in real life are marked in the map (figure 11) below.

Above is our **optimal scheme**.

5.1.5 Do the Sensitivity Analysis

Next, we will do the sensitivity analysis to the assessment model we built. The steps are shown as follows.

- Do adjustment to the weight of each factor in the assessment model.
- the treated weight of factors are still using 0-1 integer programming with genetic algorithm to find the optimal solution. that is the optimal solution of the control group
- the optimal scheme is compared with the optimal scheme under the original weight and the difference is tested to verify the sensitivity. That is to get the optimal control group.
- Compare the optimal scheme and the optimal scheme under the original weight and test the difference. Verify the sensitivity.
- We found that the original scheme is still a good one.

This table above shows the data whose weight vector is (0.7442, 0.1705,0.0852)

The data of other control groups are shown in appendix.

We found that the original scheme is still a good one.

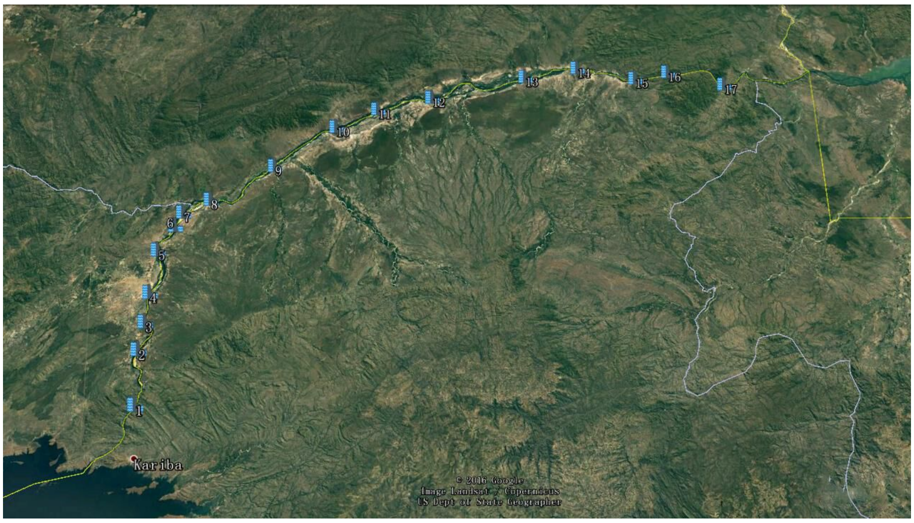

Figure 11: the Real Positions of Dams

Table 18: the Determine of the Control Groupu

	I	II	III	IV	V	VI	VII	VIII	IX	X
Status	0	1	1	0	1	1	1	1	0	0
	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Status	0	1	0	0	1	1	0	1	0	0
	XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII	XXIX	XXX
Status	1	0	1	0	0	1	1	0	0	1

5.2 Guidance for Different Conditions

In the practical application of the dam, the response measures is different under the specific condition. We will discuss about three conditions : **normal, flood and drought**.

5.2.1 Guidance for Normal Condition

In the case of no flood and drought, the dams are used normally. Close the dams when it is necessary to store water and open the dams when it is necessary to discharge the water. For the situation is too complex and changeable to talk about one by one and the usage of the dams normally is easy. So there's no further explanation.

5.2.2 Guidance for Flood Condition(Dam-break Model)

We will give a strategy under the circumstance of flood by the **dam-break model**. First of all, the two factors that influenced model are given below.

We begin to build an **instantaneous dam-break model** to discuss the dam under the flood from now.

We can use the **Saint Venant equations** to described the water wave propagation process from forward and reverse surge. This is a system of **first order hyperbolic partial differential equations**. Saint Venant equations proposed by French scientist, *Adhémar Jean Claude Barré*

de Saint − *Venant* in 1871. They are used to describe the movement of unsteady flow:

$$\begin{cases} \frac{1}{g}\frac{\partial u}{\partial t} + \frac{u}{g}\frac{\partial u}{\partial x} + \frac{\partial d}{\partial x} + (S_f - S_0) = 0 \\ \frac{\partial d}{\partial t} + \frac{\partial du}{\partial x} = \frac{\partial d}{\partial t} + d\frac{\partial u}{\partial x} + u\frac{\partial d}{\partial x} = 0 \end{cases} \tag{26}$$

Among them, u is the average velocity of water waves. d is the depth of water. x is the direction of propagation. t is time. g is the acceleration of gravity. S_f is the slope of friction. S_0 is the slope of riverbed.

The first equation reflects the law of conservation of momentum equation. The first part $\frac{1}{g}\frac{\partial u}{\partial t}$ describe the local acceleration of a wave at a position.The second part $\frac{u}{g}\frac{\partial u}{\partial x}$ describe the convective acceleration due to the uneven distribution of velocity. The first and second part are called inertia terms. The third part $\frac{\partial d}{\partial x}$ is the pressure difference caused by water depth, called water surface slope. The forth part $(S_f - S_0)$ describe the friction loss of internal friction and boundary. The equation shows that the acceleration comes from the energy loss from water's overcoming the inertia force and the friction caused by combined action of gravity and pressure.

The second equation reflects the law of Continuous quality. The first part $\frac{\partial d}{\partial t}$ describe the changes of water storage capacity. The second part $\frac{\partial du}{\partial x}$ describe the Changes of water quantity along the way.

The focus of the model is describing the decline of water waves. When the peak flow is weakened, the initial peak flow has no affect on the final result. That make the model can estimate the results for any scale of water waves.

From the dam-break model, Saint-Venant Equations, we can know that when the flood is coming, there is a linear relationship between the force that have a great impact on the degree of dam safety and the slope of riverbed. That means the impact will be larger in general.

Table 19: Slope of 17 Points

	I	II	III	IV	V	VI	VII	VIII	XII
Slope	0.91	0.34	0.36	0.21	0.13	0.22	1.03	0.25	0.48
	XV	XVI	XVIII	XXI	XXIII	XXVI	XXVII	XXX	
Slope	0.19	0.00	0.35	0.19	0.12	0.03	0.20	0.56	

Table 20: Status of 17 Points

	I	II	III	IV	V	VI	VII	VIII	XII
Status	1	1	1	1	0	1	1	1	1
	XV	XVI	XVIII	XXI	XXIII	XXVI	XXVII	XXX	
Status	1	0	1	1	0	0	1	1	

”1” in the table above means the dam will be open and ”0” in the table means the dam will be closed.

From the equation and the table x, we choose thirteen points of the seventeen points. **The thirteen dams** will be closed when there is flood.

5.2.3 Guidance for Drought Condition

Similarly, when it is drought, the use of dams is the opposite of that is flood. The strategy is to close the two dams that are opened in flood. For they are in the area that the water is urgent, closing is to prevent the water loss.

5.3 Guidance for Extreme Water

According to the data from the ZRA, we estimate the maximum average water flow is 2700 m^3/s and the minimum average water flow is 200 m^3/s. With the flow we got before, we can get the maximum and minimum flow of the seventeen position of dams.Figure x shows the maximum and the minimum flow directly.

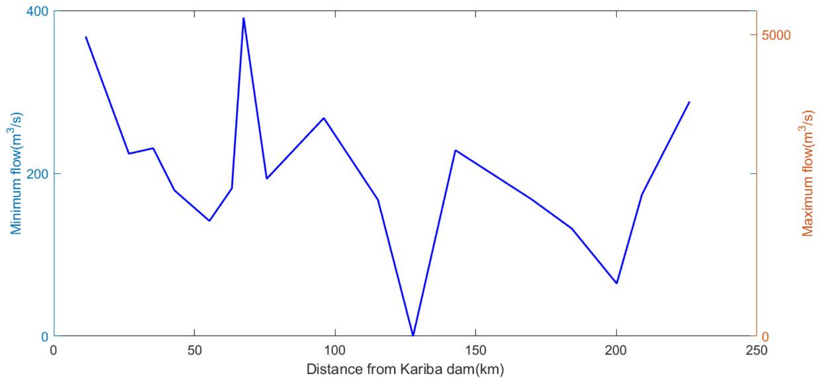

Figure 12: the Maximum and Minimum Flow

Establish the **adjustment model**. ΔQ_i is regulating flow. ΔQ_{i_2} is demanding flow. Q_{i_1} is beginning flow. ΔQ_{i-1} is regulating flow of the former dam.

$$\Delta Q_i = Q_{i_2} - Q_{i_1} - \Delta Q_{i-1} \ (i = 1,2,3...17)$$

5.3.1 Maximum Expected Discharges

Taking into account the minimum water flow, water flow should be divided in the river uniformly. So the ideal flow of each point of the dam is 200 m^3/s. According to previous models, We can obtain the regulating flow of each dam point.

5.3.2 Minimum Expected Discharges

When the value of total water flow gets maximum, the water flow of each site should be as small as possible. Considering the actual situation, the water flow of each dam should be less than 2000m^3/s. According to the information, each of the small dam should be less than 2000 m^3/s. So when the required water flow is more than 2000m^3/s, the spare water flow will be stored in the small reservoirs. When the level of the water declined to the normal line, the water flow will be released. The data is shown in the table x above.

5.4 Restrictions for Different Conditions

Considering the results from the model above, the effect of the minimum water flow on each reach of the river is not significant. When the amount of water flow reaches the largest, the water storage capacity at the dam site $VIII$ reaches the largest. Therefore, the largest impact of the dam is the area(river) around. We assume that the duration of maximum water capacity is t_0. In order to make the water flow return to the normal level, the drainage time of the entire multi-dam system $t = 4.95t_0$.

6 Conclusions

The model considers the various goals such as safety and economy, and make a comprehensive assessment of the solution. We analyze the problem based on the literature, so the judgement is more reliable. By the sensitivity analysis, as the weights change, our solution shows great range of application.

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